

Wireless Networks

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Abstract

This paper examines capacity scaling in dense wireless fading networks, where L single-antenna source-destination terminal pairs communicate concurrently through a common set of K single-antenna relay terminals using two-hop half-duplex relaying. In the perfectly synchronized case, assuming perfect channel state information (CSI) at the relays (coherent network), the network capacity is known to scale (asymptotically in K for fixed L) as $C = \frac{L}{2} \log(K) + O(1)$ [1]. Moreover, relay partitioning as described in [1] and matched-filtering at the relay terminals achieves network capacity with independent (coherent) decoding at the destination terminals. On a conceptual level this result implies that coherent relaying orthogonalizes the effective multiple-input multiple-output (MIMO) channel between the source and the destination terminals in a distributed fashion, hence the name “distributed orthogonalization”. In the absence of CSI at the relay terminals (noncoherent network), it was shown in [1] that a simple amplify-and-forward protocol, asymptotically in K for fixed L , turns the network into a point-to-point MIMO link with high-SNR capacity $C = \frac{L}{2} \log(\text{SNR}) + O(1)$.

The results in [1] were derived under the idealistic assumptions of perfect synchronization, perfect CSI at the relays and frequency-flat fading. The purpose of this paper is to relax these assumptions and study the resulting impact on the scaling laws in [1] assuming single-antenna terminals. Our contributions can be summarized as follows:

- We demonstrate that lack of synchronization in the network, under quite general conditions on the synchronization error characteristics, leaves the capacity scaling laws for both coherent and

noncoherent networks fundamentally unchanged. However, synchronization errors do result in a reduction of the effective SNR at the destination terminals which amounts to an increased number of relay terminals needed for a given network capacity requirement. Quantitative results for this SNR degradation, as a function of the synchronization error characteristics, are provided.

- For coherent and noncoherent networks, we show that the presence of frequency-selective fading does not have an impact on the capacity scaling laws provided the relaying strategies take the frequency-selective nature of the individual channels explicitly into account. Corresponding relaying strategies are proposed.
- For coherent networks, we analyze the impact of imperfect CSI at the relays on the corresponding network capacity scaling law. It is demonstrated that distributed orthogonalization can be obtained if a minimum amount of coherence at the relays can be sustained.

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I. INTRODUCTION

We consider a wireless fading relay network where L single-antenna source-destination terminal pairs communicate concurrently and over the same frequency-band through two-hop half-duplex relaying over a common set of K single-antenna relay terminals (see Fig. 1). For a perfectly synchronized network with fixed L , letting $K \rightarrow \infty$, the following results have been established in [1] under the dense network assumption with frequency-flat fading links:

- In the presence of perfect channel state information (CSI) at the relay terminals (coherent network), the network capacity scales as $C = \frac{L}{2} \log(K) + O(1)$ and can be achieved by relay partitioning, matched-filtering at the relay terminals and independent (coherent) decoding at the destination terminals. This result implies that *distributed array gain* [2] and *spatial multiplexing gain* [3], [4], [2] can be obtained in a completely distributed fashion, i.e., without cooperation between any of the terminals; the corresponding effect is referred to as *distributed orthogonalization*.

- In the absence of CSI at the relay terminals (noncoherent network), a simple amplify-and-forward (AF) strategy turns the relay network into a point-to-point multiple-input multiple-output (MIMO) link. Provided that the destination terminals have perfect knowledge of the resulting effective MIMO channel, joint (across destination terminals) decoding achieves a high-SNR capacity¹ of $C = \frac{L}{2} \log(\text{SNR}) + O(1)$ and hence full (up to the factor 1/2-loss due to half-duplex communication) spatial multiplexing gain.

Contributions. The results in [1] were derived under the idealistic assumptions of perfect synchronization, perfect CSI at the relays (for coherent networks) and frequency-flat fading. In practice, however, perfect synchronization across geographically distributed network terminals as well as perfect CSI at the relays are difficult to realize. Moreover, in wideband systems, the individual links in the network will exhibit frequency-selective fading. The purpose of this paper is to relax the idealistic assumptions made in [1] and study the resulting impact on the capacity scaling laws for coherent and noncoherent networks with single-antenna terminals. In the coherent network case, we shall furthermore investigate the robustness of distributed orthogonalization with respect to (w.r.t.) lack of synchronization, imperfect CSI at the relays and frequency-selective fading. Our detailed contributions can be summarized as follows:

- We show that under quite general conditions on the timing offset characteristics and CSI at the relays (in the coherent case), the scaling law in both the coherent and noncoherent network cases remains fundamentally unchanged. Moreover, in the coherent network case independent decoding at the destination terminals continues to achieve network capacity in the large K limit, which implies that the distributed orthogonalization effect is very robust w.r.t. synchronization errors and CSI at the relays. However, the presence of timing errors and imperfect CSI at the relays (in the coherent network case) does result in a reduction of the effective signal-to-noise ratio (SNR) at the destination terminals. Equivalently, in

¹Here, the $O(1)$ -notation is with respect to SNR.

the asynchronous case and/or for imperfect CSI at the relays a larger number of relays is needed to achieve the same network capacity as in the perfectly synchronized and/or perfect relay CSI case. We quantify the corresponding losses as a function of the timing offset characteristics and the CSI accuracy at the relays.

- We extend the results in [1] (for the case of single-antenna terminals) to the case of frequency-selective fading links. It is demonstrated that the same scaling laws as in the frequency-flat fading case can be achieved provided the relaying strategies take the frequency-selective nature of the individual links explicitly into account. Corresponding relaying strategies are proposed. In the coherent network case it is furthermore demonstrated that distributed orthogonalization is possible with a minimum amount of CSI (knowledge of as little as one tap of the assigned channels is sufficient) at the relays. In the noncoherent case AF is shown to turn the relay network into a frequency-selective fading point-to-point MIMO link.

Relation to previous work. Beginning with the pioneering work of Gupta and Kumar [5] and Gastpar and Vetterli [6], studying the ultimate performance limits of wireless networks through their asymptotic capacity scaling behavior has been attracting increasing interest [7], [8], [9], [10], [11], [12], [1], [13], [14]. In finite size relay networks distributed space-time code design and information-theoretic performance limits for the case of single-antenna nodes have recently been studied in [15], [16], [17], [18], [19]. Capacity results for MIMO relay channels with a finite number of relays can be found in [20], [21], [18]. For networks with fixed links (e.g. wired networks) the concept of network coding has recently attracted significant interest [22], [23].

In [8], accounting for synchronization errors through multiplicative phase terms in the complex-valued channel gains, it is concluded that modest synchronization errors leave the power efficiency scaling law in coherent networks (under more stringent requirements on CSI at the relays than in [1]) unchanged. The concept of distributed orthogonalization was proposed in [1] and, as shown in [24], without making it explicit previously in [8]. The protocol described in [8]

realizes distributed orthogonalization but requires that each of the relay terminals knows all the links between the source terminals and the relay (backward channels) and between the relay and all destination terminals (forward channels). In contrast, the protocol in [1] is based on relay partitioning and requires knowledge of only one backward and forward channel per relay. Results on the transport capacity of fading networks have been reported in [12]. The impact of frequency-selective fading and imperfect CSI on fading relay network capacity scaling does not seem to have been studied before.

Notation. The superscripts T , H and $*$ stand for transposition, conjugate transpose, and element-wise conjugation, respectively. For an $m \times n$ matrix $\mathbf{A} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n]$, we define the $mn \times 1$ vector $\text{vec}(\mathbf{A}) = [\mathbf{a}_1^T \ \mathbf{a}_2^T \ \dots \ \mathbf{a}_n^T]^T$. \mathbf{I}_m stands for the $m \times m$ identity matrix. \mathcal{E} denotes the expectation operator. A circularly symmetric complex Gaussian random variable (RV) is a RV $Z = X + jY \sim \mathcal{CN}(0, N_0)$, where X and Y are i.i.d. $\mathcal{N}(0, N_0/2)$. \mathbf{H}_w denotes a matrix of appropriate size consisting of i.i.d. $\mathcal{CN}(0, 1)$ entries. The notation $u(x) = O(v(x))$ denotes that $|u(x)/v(x)|$ remains bounded as $x \rightarrow \infty$. $|\mathcal{X}|$ stands for the cardinality of the set \mathcal{X} . Throughout the paper all logarithms are to the base 2.

Organization of the paper. The rest of this paper is organized as follows. Section II introduces the channel and signal models for the asynchronous relay network assuming frequency-flat fading links. In Section III, we briefly review the (perfect synchronization) capacity scaling results reported in [1] for the case of single-antenna terminals. Section IV contains the capacity scaling results for the coherent and noncoherent asynchronous cases and quantifies the impact of imperfect CSI and synchronization errors on capacity scaling and distributed orthogonalization. In Section V, we present the channel and signal models for relay networks with frequency-selective fading links followed by the corresponding results on capacity scaling and distributed orthogonalization along with the corresponding relaying strategies. Numerical results are presented in Section VI. We conclude in Section VII.

II. CHANNEL AND SIGNAL MODEL

In this section, we introduce the basic setup and describe the channel and signal model for the asynchronous frequency-flat fading case. The modifications to the channel and signal model required to take into account frequency-selective fading are presented in Section V.

General assumptions. We consider an asynchronous wireless network consisting of $K + 2L$ single-antenna terminals with L designated source-destination terminal pairs $\{\mathcal{S}_l, \mathcal{D}_l\}$ ($l = 1, 2, \dots, L$) and K relay terminals \mathcal{R}_k ($k = 1, 2, \dots, K$). Source terminal \mathcal{S}_l intends to communicate solely with destination terminal \mathcal{D}_l , a “dead-zone” of non-zero radius around each \mathcal{S}_l and \mathcal{D}_l is free of relay terminals, no direct links exist between the individual source-destination terminal pairs (caused for instance by large separation or heavy shadowing), and transmission takes place over two time slots using two-hop relaying. None of the terminals can transmit and receive simultaneously. In the first time slot, the source terminals \mathcal{S}_l broadcast their information to all relay terminals over one symbol period of duration T_s . After processing the received signals, in the second time slot the relay terminals broadcast the processed data to all destination terminals over one symbol period (of duration T_s) while the source terminals are silent. In order to accommodate propagation delay differences between the network terminals, we furthermore introduce guard periods so that the entire time duration spanned by the first (second) time slot is Δ_1 (Δ_2). We take $\Delta_1 > T_s$ and $\Delta_2 > T_s$ large enough to ensure that over the first hop each relay terminal receives all of the L data symbols of duration T_s entirely and likewise, over the second hop, each destination terminal receives each relay transmission entirely. Finally, we assume that all terminals are located within a domain of fixed area (dense network assumption) with independently chosen random locations. The dense network assumption becomes crucial in Sections III-VI when studying the large number of relay terminals capacity behavior of the network.

Channel and signal model. In the following, we establish the channel and signal model for a single end-to-end use of the relay network over the time period $0 \leq t \leq \Delta_1 + \Delta_2$. For the sake of simplicity of exposition, we assume that all source transmissions are perfectly coordinated to begin at $t = 0$, while the relay transmissions are perfectly coordinated to begin at $t = \Delta_1$. In practice, these assumptions require a common clock for the \mathcal{S}_l and a common clock for the \mathcal{R}_k . The more general case accounting for complete lack of coordination in the network can easily be dealt with, but requires tedious (however straightforward) modifications of the derivations in Sections IV and V.

We assume that starting at $t = 0$ the l -th source terminal transmits

$$s_l(t) = s_l u(t) e^{j\omega_c t}, \quad l = 1, 2, \dots, L \quad (1)$$

where the s_l are i.i.d. $\mathcal{CN}(0, 1)$ data symbols, $u(t)$ compactly supported in $[0, T_s]$ is the real-valued baseband pulse-shaping function normalized according to $\int_0^{T_s} u^2(t) dt = 1$ and ω_c denotes the carrier frequency (in rad/s) of the modulated waveform. Note that all source terminals employ the same pulse-shaping function $u(t)$ and operate at the same carrier frequency ω_c .

All channels in the network are independently frequency-flat block fading [25] with the same block length, and with independent realizations across blocks (the block duration is taken to be an integer multiple of $\Delta_1 + \Delta_2$). The case of frequency-selective fading channels will be dealt with in Section V.

The signal received at the k -th relay terminal during the first time slot is given by

$$r_k(t) = \sum_{l=1}^L \sqrt{E_{k,l}} h_{k,l} s_l(t - \tau_{k,l}) + n_k(t), \quad k = 1, 2, \dots, K \quad (2)$$

where $E_{k,l}$ is the average energy received at \mathcal{R}_k through the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ link² (having accounted for path loss and shadowing), $h_{k,l}$ and $\tau_{k,l}$ denote the corresponding $\mathcal{CN}(0, 1)$ backward channel gain (accounting for small-scale fading) and propagation delay, respectively, and $n_k(t)$ is additive

² $\mathcal{A} \rightarrow \mathcal{B}$ signifies communication from terminal \mathcal{A} to terminal \mathcal{B} .

white Gaussian noise (analytic representation) at the k -th relay terminal (independent across k) with power spectral density [26]

$$S(\omega) = \begin{cases} N_o, & \omega > 0 \\ \frac{N_o}{2}, & \omega = 0 \\ 0, & \omega < 0 \end{cases} . \quad (3)$$

Each relay terminal now demodulates the received signal through matched-filtering (MF) according to

$$r_k = \int_{\theta_k}^{\theta_k + T_s} r_k(t) u(t - \theta_k) e^{-j\omega_c(t - \theta_k)} dt, \quad k = 1, 2, \dots, K \quad (4)$$

where $0 \leq \theta_k \leq \Delta_1 - T_s$ ($k = 1, 2, \dots, K$) is the demodulator start-time (can be chosen randomly or based on (partial) synchronization information) for the k -th relay terminal. Combining (1), (2), and (4), we have

$$r_k = \sum_{l=1}^L \left(\underbrace{\sqrt{E_{k,l}} h_{k,l} e^{j\omega_c \xi_{k,l}} \int_0^{T_s} u(t) u(t + \xi_{k,l}) dt}_{\bar{h}_{k,l}} s_l \right) + n_k \quad (5)$$

where $\xi_{k,l} = \theta_k - \tau_{k,l}$ and $\bar{h}_{k,l}$ denote the timing offset (synchronization error) and effective backward channel (including the effect due to the timing offset), respectively, for the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ link, and n_k is $\mathcal{CN}(0, N_o)$ noise (independent across k). In the following, we assume that the timing offsets $\xi_{k,l}$ are i.i.d. random variables³ (independent across k and l). The k -th ($k = 1, 2, \dots, K$) relay terminal processes r_k to produce a unit average⁴ energy output t_k (i.e., $\mathcal{E}\{|t_k|^2\} = 1$), which is then modulated to form the signal $q_k(t)$ transmitted during the second time slot and given by

$$q_k(t) = t_k u(t - \Delta_1) e^{j\omega_c(t - \Delta_1)} \quad (6)$$

³The i.i.d. assumption on the timing offsets is made for simplicity of exposition only and is not conceptual.

⁴Averaged over the backward channels $\bar{h}_{k,l}$, data symbols s_l , and noise n_k .

with $u(t)$ and ω_c as defined in (1). The signal received at the l -th destination terminal is given by

$$y_l(t) = \sum_{k=1}^K \sqrt{P_{l,k}} g_{l,k} q_k(t - \delta_{l,k}) + z_l(t), \quad l = 1, 2, \dots, L \quad (7)$$

where $P_{l,k}$ denotes the average energy received at the l -th destination terminal through the $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link (having accounted for path loss and shadowing), $g_{l,k}$ and $\delta_{l,k}$ represent the corresponding $\mathcal{CN}(0, 1)$ channel gain (accounting for small-scale fading) and propagation delay, respectively, and $z_l(t)$ is additive white Gaussian noise (analytic representation) at the l -th destination terminal (independent across l) with power spectral density specified in (3).

Finally, each destination terminal demodulates its received signal according to

$$y_l = \int_{\Delta_1 + \phi_l}^{\Delta_1 + \phi_l + T_s} y_l(t) u(t - \Delta_1 - \phi_l) e^{-j\omega_c(t - \Delta_1 - \phi_l)} dt, \quad l = 1, 2, \dots, L \quad (8)$$

where $\Delta_1 + \phi_l$ is the demodulator start-time (ϕ_l can be chosen randomly or based on (partial) synchronization information) at the l -th destination terminal and $0 \leq \phi_l \leq \Delta_2 - T_s$. Combining (6), (7), and (8), we get

$$y_l = \sum_{k=1}^K \left(\underbrace{\sqrt{P_{l,k}} g_{l,k} e^{j\omega_c \eta_{l,k}} \int_0^{T_s} u(t) u(t + \eta_{l,k}) dt}_{\bar{g}_{l,k}} b_k \right) + z_l \quad (9)$$

where $\eta_{l,k} = \phi_l - \delta_{l,k}$ and $\bar{g}_{l,k}$ represent the timing offset and effective forward channel (including the effect due to the timing offset), respectively, for the $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link, and z_l is $\mathcal{CN}(0, N_o)$ noise (independent across l). Just as for the first time slot, we again assume that the timing offsets $\eta_{l,k}$ are i.i.d. RVs⁵, independent of the $\xi_{k,l}$.

As already mentioned above, throughout the paper, path loss and shadowing statistics (i.e., large-scale fading) are captured by the $E_{k,l}$ ($k = 1, 2, \dots, K$, $l = 1, 2, \dots, L$) (for the first hop) and the $P_{l,k}$ ($l = 1, 2, \dots, L$, $k = 1, 2, \dots, K$) (for the second hop). We assume that these parameters are i.i.d. RVs, strictly positive and bounded. The exact statistics of the $E_{k,l}$ and $P_{l,k}$ will in general

⁵Again, the i.i.d. assumption on the timing offsets is made for simplicity of exposition only and is not conceptual.

depend on the network topology. The randomness of the $E_{k,l}$ and $P_{l,k}$ reflects the fact that the terminal locations are chosen randomly, strict positivity is a consequence of the assumption that the domain under consideration is of fixed area, and the dead-zone assumption implies that the parameters $E_{k,l}$ and $P_{l,k}$ are bounded. If the relay terminals are located randomly and uniformly within the domain of interest the independence assumption on the set $\{E_{k,l}\}$ and the set $\{P_{l,k}\}$ is well justified. If the relay terminals are, however, confined to certain geographic areas (e.g., only at cell edge or only within a certain range from the source and destination terminals) this has to be accounted for by allowing the elements in the sets $\{E_{k,l}\}$ and $\{P_{l,k}\}$ to have different distributions (in particular different means). Relaxing the i.i.d. assumption on $\{E_{k,l}\}$ and $\{P_{l,k}\}$ leads to more stringent requirements on the amount of CSI at the destination terminals, but leaves the network capacity scaling laws unchanged. For the perfectly synchronized case this was demonstrated in [1]. For the sake of simplicity of exposition, in this paper, we shall therefore formulate our results for the i.i.d. case only. Furthermore, the $E_{k,l}$, $\xi_{k,l}$, $P_{l,k}$ and $\eta_{l,k}$ are assumed to remain constant over the entire time period of interest (spanning multiple uses of the network). Finally, the $\xi_{k,l}$ and the $\eta_{l,k}$ are independent of $h_{k,l}$, $E_{k,l}$, $g_{l,k}$, $P_{l,k}$ $\forall l, k$ and all noise terms.

III. BRIEF REVIEW OF SCALING LAWS IN THE SYNCHRONOUS CASE

In this section, in order to set the stage for Section IV, we shall next specialize the results on coherent and noncoherent (under AF) relaying in [1] to the single-antenna terminal case assuming perfect synchronization, i.e., $\xi_{k,l}$ and $\eta_{l,k}$ are deterministic and satisfy $\xi_{k,l} = 0$, $\eta_{l,k} = 0$ $\forall l, k$ and $\Delta_1 = \Delta_2 = T_s$ with a rectangular pulse shaping function $u(t) = 1$ for $t \in [0, T_s]$ and $u(t) = 0$ otherwise.

A. The Coherent Network Case

We start with the *coherent network case* where the k -th relay terminal has perfect knowledge of the individual channels corresponding to the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}_l$ ($l = 1, 2, \dots, L$) links as well as the path loss and shadowing factors $E_{k,l}$ and $P_{l,k}$.

Applying the *cut-set theorem* [27, Th. 14.10.1] by separating the source terminals \mathcal{S}_l ($l = 1, 2, \dots, L$) from the rest of the network (broadcast cut), assuming that L is fixed and $K \rightarrow \infty$, the network (sum) capacity is upper bounded by [1]

$$C_u = \frac{L}{2} \log(K) + O(1). \quad (10)$$

This upper bound is achieved when *all relay terminals can fully cooperate* and can furthermore convey the relays' transmit signals b_k in a lossless fashion to the *cooperating destination terminals*. The factor $1/2$ -loss in the pre-log is due to the fact that communication takes place over two time slots and the source terminals transmit only in the first time slot.

A protocol which achieves (10) (up to the $O(1)$ term) is described in detail in [1] and can briefly be summarized as follows. The relay terminals are partitioned into L subsets \mathcal{X}_l ($l = 1, 2, \dots, L$) each of which is assigned to one of the L source-destination terminal pairs. The relaying strategy for $\mathcal{R}_k \in \mathcal{X}_l$ is as follows. The received signal r_k is matched-filtered w.r.t. the (backward) channel $\mathcal{S}_l \rightarrow \mathcal{R}_k$ followed by matched-filtering w.r.t. the (forward) channel $\mathcal{R}_k \rightarrow \mathcal{D}_l$ subject to energy normalization so that $\mathcal{E}\{|t_k|^2\} = 1$. When the number of relay terminals $K \rightarrow \infty$ we need to ensure that $|\mathcal{X}_l| \rightarrow \infty$ ($l = 1, 2, \dots, L$) so that each source-destination terminal pair gets served by an infinite number of relay terminals. The l -th destination terminal \mathcal{D}_l is assumed to have perfect knowledge of the effective scalar channels between the \mathcal{S}_i ($i = 1, 2, \dots, L$) and \mathcal{D}_l and the destination terminals perform independent decoding, i.e., there is no cooperation between the \mathcal{D}_l ($l = 1, 2, \dots, L$). In summary, we can conclude that the protocol described above realizes a spatial multiplexing gain of $\frac{L}{2}$ (pre-log in (10)) with each of the multiplexed data streams experiencing a distributed array gain of K (factor inside the logarithm) while each of the relay terminals needs to know only one backward and one forward channel. The significance of this result lies in the fact that the (cooperative) cut-set upper bound can be achieved (up to the $O(1)$ -term) without cooperation between the destination terminals. *Multi-stream interference* is therefore eliminated in a fully distributed fashion or equivalently the effective MIMO channel

between the \mathcal{S}_l and the \mathcal{D}_l is *orthogonalized in a distributed fashion*. The corresponding effect is termed *distributed orthogonalization* [1]. The protocol proposed in [8] was shown in [24] to perform distributed orthogonalization as well. Relay partitioning is not required in the protocol in [8], but the k -th relay needs to know all its backward channels $\mathcal{S}_l \rightarrow \mathcal{R}_k$ ($l = 1, 2, \dots, L$) and all its forward channels $\mathcal{R}_k \rightarrow \mathcal{D}_l$ ($l = 1, 2, \dots, L$). In the remainder of this paper, in the coherent network case, we shall focus on the protocol introduced in [1].

B. The Noncoherent Case

The MF-based protocol described in the previous subsection requires CSI at the relay terminals. If this assumption is relaxed and each relay terminal simply amplifies-and-forwards its received signal subject to the power constraint $\mathcal{E}\{|t_k|^2\} = 1$, for fixed L and $K \rightarrow \infty$, the network is turned into a point-to-point MIMO Rayleigh fading link with Gaussian noise and high-SNR capacity⁶ given by [1]

$$C = \frac{L}{2} \log(\text{SNR}) + O(1). \quad (11)$$

Here, SNR is an effective signal-to-noise-ratio depending on the statistics of the $E_{k,l}$ and $P_{l,k}$ and on N_0 . The exact expression for SNR in (11) is provided in [1]. In contrast to the coherent case described in Section III.A, achieving (11) requires that the destination terminals cooperate and jointly decode the $L \times 1$ vector signal transmitted by the \mathcal{S}_l . Moreover, perfect knowledge of the composite $L \times L$ MIMO channel is required at the destination terminals.

IV. CAPACITY SCALING LAWS IN THE ASYNCHRONOUS AND IMPERFECT CSI CASES

In this section, we study the impact of lack of synchronization on the results summarized in Section III. In order to simplify the exposition, we introduce the notation

$$\bar{h}_{k,l} = h_{k,l} f(\xi_{k,l}) \quad (12)$$

$$\bar{g}_{l,k} = g_{l,k} p(\eta_{l,k}) \quad (13)$$

⁶Note that here the $O(1)$ -notation is w.r.t. SNR.

where it follows from (5) and (9) that $f(\xi_{k,l}) = e^{j\omega_c \xi_{k,l}} \int_0^{T_s} u(t)u(t + \xi_{k,l}) dt$ and $p(\eta_{l,k}) = e^{j\omega_c \eta_{l,k}} \int_0^{T_s} u(t)u(t + \eta_{l,k}) dt$. Since the $\xi_{k,l}$ and $\eta_{l,k}$ are i.i.d. RVs it follows that the quantities $f(\xi_{k,l})$ and $p(\eta_{l,k})$ will be i.i.d. RVs as well with probability density functions (pdfs) depending on the pdfs of $\xi_{k,l}$ and $\eta_{l,k}$, respectively, as well as the pulse-shaping function $u(t)$.

A. The Coherent Network Case

As in the synchronous case described in Section III.A, we partition the relay terminals into L subsets each of which is assigned to one of the L source-destination terminal pairs. Two different scenarios are considered.

Scenario 1: Relay terminal $\mathcal{R}_k \in \mathcal{X}_l$ performs matched-filtering of r_k w.r.t. the effective (including the effect of the timing offset) backward channel $\bar{h}_{k,l}$ and the effective forward channel $\bar{g}_{l,k}$ with appropriate normalization to meet the average per-relay transmit power constraint. This setup corresponds to a scenario where the backward and forward channels, including the timing offsets, are learned (i.e., estimated) and the network operates in an asynchronous mode. Subsequently, this mode of operation will be called the “synchronization error compensated matched-filtering mode”.

Theorem 1. *For an asynchronous relay network operating in the synchronization error compensated matched-filtering mode, assuming a fixed number of source-destination terminal pairs L , in the large relay limit $K \rightarrow \infty$ such that⁷ $|\mathcal{X}_1| = |\mathcal{X}_2| = \dots = |\mathcal{X}_L| = K/L$, the network capacity scales as*

$$C = \frac{LT_s}{\Delta_1 + \Delta_2} \log(K) + O(1) \quad (14)$$

⁷Like in [1] the assumptions of equal size relay clusters and K being increased in integer multiples of L are made for the sake of simplicity of exposition only. As long as $|\mathcal{X}_l| \rightarrow \infty$ ($l = 1, 2, \dots, L$) for $K \rightarrow \infty$ the scaling result (14) is valid.

if⁸

$$\mathcal{E}\{|f(\xi)|\} > 0 \quad \text{and} \quad \mathcal{E}\{|p(\eta)|\} > 0. \quad (15)$$

Moreover, network capacity is achieved with independent decoding at the destination terminals assuming that each \mathcal{D}_l has perfect knowledge of the effective scalar channels between the \mathcal{S}_i ($i = 1, 2, \dots, L$) and \mathcal{D}_l .

Proof: A major part of this proof follows the proof of Theorem 2 (with slight modifications taking into account that lack of synchronization leads to effective channels different from the physical channels) in [1]. We shall therefore summarize the key steps of the proof only, focus on the differences to [1] due to synchronization errors, emphasize the aspects needed for the formulation of our remaining results on coherent networks and refer the interested reader to [1] for details.

We start by computing a lower bound on network capacity which is obtained by simply evaluating the network capacity achieved by relay partitioning, matched-filtering at the relays and independent decoding at the destination terminals. Consider a relay terminal $\mathcal{R}_k \in \mathcal{X}_l$. Since the relay matched-filters to the effective backward channel $\bar{h}_{k,l}$ and the effective forward channel $\bar{g}_{l,k}$, the unit average energy processed data signal to be transmitted is given by

$$t_k = \frac{r_k \bar{h}_{k,l}^* \bar{g}_{l,k}^*}{|\bar{g}_{l,k}| \sqrt{2E_{k,l} |f(\xi_{k,l})|^4 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 |f(\xi_{k,l})|^2 + N_0 |f(\xi_{k,l})|^2}} \quad (16)$$

Now, the demodulated signal at \mathcal{D}_l , is obtained as

$$y_l = h_l^{sig} s_l + \sum_{j=1, j \neq l}^L h_{l,j}^{int} s_j + N_l \quad (17)$$

where h_l^{sig} denotes the effective scalar channel between the l -th source-destination terminal pair, $h_{l,j}^{int}$ stands for the effective interference channel between the j -th source terminal ($j \neq l$) and

⁸Note that due to the i.i.d. assumption on the sets $\{\xi_{k,l}\}$ and $\{\eta_{l,k}\}$, the expectations $\mathcal{E}\{|f(\xi_{k,l})|\}$ and $\mathcal{E}\{|p(\eta_{l,k})|\}$ do not depend on the indices k, l .

the l -th destination terminal, and N_l is effective noise consisting of noise forwarded by the relays and thermal noise at the receiver.

It follows from (17) that

$$h_l^{sig} = \sum_{k, \mathcal{R}_k \in \mathcal{X}_l} d_{k,l} + \sum_{m=1, m \neq l}^L \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_m} f_{k,l,m} \right)$$

where

$$d_{k,l} = \frac{\sqrt{P_{l,k}} \sqrt{E_{k,l}} |g_{l,k}| |p(\eta_{l,k})| |h_{k,l}|^2 |f(\xi_{k,l})|}{\sqrt{2E_{k,l} |f(\xi_{k,l})|^2 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 + N_0}} \quad (18)$$

and

$$f_{k,l,m} = \frac{\sqrt{P_{l,k}} \sqrt{E_{k,l}} \bar{g}_{l,k} \bar{g}_{m,k}^* \bar{h}_{k,m}^* \bar{h}_{k,l}}{|\bar{g}_{m,k}| \sqrt{2E_{k,m} |f(\xi_{k,m})|^4 + \sum_{i=1, i \neq m}^L E_{k,i} |f(\xi_{k,i})|^2 |f(\xi_{k,m})|^2 + N_0 |f(\xi_{k,m})|^2}}.$$

Similary, from (17) we obtain

$$h_{l,j}^{int} = \sum_{k, \mathcal{R}_k \in \mathcal{X}_l} x_{k,l,j} + \sum_{k, \mathcal{R}_k \in \mathcal{X}_j} w_{k,l,j} + \sum_{m=1, m \neq l,j}^L \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_m} v_{k,l,j,m} \right)$$

where

$$x_{k,l,j} = \frac{\sqrt{P_{l,k}} \sqrt{E_{k,j}} |\bar{g}_{l,k}| \bar{h}_{k,l}^* \bar{h}_{k,j}}{\sqrt{2E_{k,l} |f(\xi_{k,l})|^4 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 |f(\xi_{k,l})|^2 + N_0 |f(\xi_{k,l})|^2}}$$

$$w_{k,l,j} = \frac{\sqrt{P_{l,k}} \sqrt{E_{k,j}} \bar{g}_{l,k} \bar{g}_{j,k}^* |h_{k,j}|^2 |f(\xi_{k,j})|}{|\bar{g}_{j,k}| \sqrt{2E_{k,j} |f(\xi_{k,j})|^2 + \sum_{i=1, i \neq j}^L E_{k,i} |f(\xi_{k,i})|^2 + N_0}}$$

$$v_{k,l,j,m} = \frac{\sqrt{P_{l,k}} \sqrt{E_{k,j}} \bar{g}_{l,k} \bar{g}_{m,k}^* \bar{h}_{k,m}^* \bar{h}_{k,j}}{|\bar{g}_{m,k}| \sqrt{2E_{k,m} |f(\xi_{k,m})|^4 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 |f(\xi_{k,m})|^2 + N_0 |f(\xi_{k,m})|^2}}.$$

Finally, we note that the effective noise term N_l may be decomposed as

$$N_l = \sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \tilde{n}_{k,l} + \sum_{m=1, m \neq l}^L \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_m} \tilde{q}_{k,l,m} \right) + z_l$$

where $\tilde{n}_{k,l} | \{ \bar{h}_{k,l}, \bar{g}_{l,k} \}_{k=1}^K \sim \mathcal{CN}(0, a_{k,l} N_0)$ and $\tilde{q}_{k,l,m} | \{ \bar{h}_{k,m}, \bar{g}_{l,k}, \bar{g}_{m,k} \}_{k=1}^K \sim \mathcal{CN}(0, b_{k,l,m} N_0)$

with

$$a_{k,l} = \frac{P_{l,k} |\bar{g}_{l,k}|^2 |h_{k,l}|^2}{2E_{k,l} |f(\xi_{k,l})|^2 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 + N_0} \quad (19)$$

and

$$b_{k,l,m} = \frac{P_{l,k} |h_{k,m}|^2 |\bar{g}_{l,k}|^2}{2E_{k,m} |f(\xi_{k,m})|^2 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 + N_0}. \quad (20)$$

Assuming independent decoding at each of the destination terminals based on perfect knowledge of h_l^{sig} and $h_{l,j}^{int}$ ($j = 1, 2, \dots, L, j \neq l$) at \mathcal{D}_l , the large⁹ K capacity achieved by relay partitioning, matched-filtering at the relays and independent decoding at the destination terminals is given by

$$C_{lower} = \sum_{l=1}^L \mathcal{E}_{h_{k,i}, g_{i,k}} \{I_l\} \quad (21)$$

where

$$I_l = \frac{T_s}{\Delta_1 + \Delta_2} \log \left(1 + \frac{|h_l^{sig}|^2}{\sum_{j=1, j \neq l}^L |h_{l,j}^{int}|^2 + N_0 \left(1 + \sum_{k, \mathcal{R}_k \in \mathcal{X}_l} a_{k,l} + \sum_{m=1, m \neq l}^L \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_m} b_{k,l,m} \right) \right)} \right) \quad (22)$$

for $l = 1, 2, \dots, L$. Next, following the same steps as in Theorem 2 in [1], we obtain

$$I_l \stackrel{\text{w.p.1}}{\rightarrow} \frac{T_s}{\Delta_1 + \Delta_2} \log \left(1 + \frac{K \delta_l^2}{LN_0 \left(\frac{L}{K} + \alpha_l + \beta_l \right)} \right) \quad (23)$$

where

$$\delta_l = \sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \frac{\mathcal{E}\{d_{k,l}\}}{K/L} = \mathcal{E}\{d_{k,l}\} \quad (24)$$

$$\alpha_l = \sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \frac{\mathcal{E}\{a_{k,l}\}}{K/L} = \mathcal{E}\{a_{k,l}\} \quad (25)$$

and

$$\beta_l = \sum_{m=1, m \neq l}^L \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_m} \frac{\mathcal{E}\{b_{k,l,m}\}}{K/L} \right) = (L-1) \mathcal{E}\{b_{k,l,m}\} \quad (26)$$

are assumed to be strictly positive and we exploited the fact that the sets $\{d_{k,l}\}$, $\{a_{k,l}\}$ and $\{b_{k,l,m}\}$ are all i.i.d. The conditions under which the quantities δ_l , α_l and β_l are strictly positive

⁹The effective noise term N_l becomes conditionally (on the h_l^{sig} and $h_{l,j}^{int}$ ($j = 1, 2, \dots, L, j \neq l$)) circularly symmetric complex Gaussian for K large. In the finite K case, we need each of the \mathcal{D}_l to know all the scalar effective channels in the network as well as all coefficients $E_{k,l}$ and $P_{l,k}$ to render the effective noise term conditionally Gaussian (cf. Theorem 2 in [1]).

will be derived shortly. The capacity achieved by relay partitioning and matched-filtering is now given by

$$C_{lower} \stackrel{\text{w.p.1}}{\rightarrow} \frac{T_s}{\Delta_1 + \Delta_2} \sum_{l=1}^L \log \left(1 + \frac{K\delta_l^2}{LN_0 \left(\frac{L}{K} + \alpha_l + \beta_l \right)} \right) = \frac{LT_s}{\Delta_1 + \Delta_2} \log(K) + O(1). \quad (27)$$

To ensure strict positivity of δ_l we require

$$\mathcal{E} \left\{ \frac{\sqrt{P_{l,k}} \sqrt{E_{k,l}} |g_{l,k}| |p(\eta_{l,k})| |h_{k,l}|^2 |f(\xi_{k,l})|}{\sqrt{2E_{k,l} |f(\xi_{k,l})|^2 + \sum_{i=1, i \neq l}^L E_{k,i} |f(\xi_{k,i})|^2 + N_0}} \right\} > 0. \quad (28)$$

Since $|f(\xi_{k,l})| \leq 1$, the $P_{l,k}$ are strictly positive and the $E_{k,l}$ are bounded $\forall k, l$, it follows that for (28) to hold it is sufficient to ensure

$$\mathcal{E}\{|f(\xi)|\} > 0 \quad \text{and} \quad \mathcal{E}\{|p(\eta)|\} > 0 \quad (29)$$

where we exploited the fact that the sets $\{\xi_{k,l}\}$ and $\{\eta_{l,k}\}$ are i.i.d. The positivity of the α_l and the β_l can be established similarly and again leads to the conditions (29).

An upper bound on asynchronous network capacity is given by the cut-set bound for the perfectly synchronized case (10) taking into account the spectral efficiency loss due to the use of guard regions and yields

$$C_{upper} \stackrel{\text{w.p.1}}{\rightarrow} \frac{LT_s}{\Delta_1 + \Delta_2} \log(K) + O(1) \quad (30)$$

where the $O(1)$ -term will in general be different from that in (27). This difference in the $O(1)$ -term is due to asynchronicity as well as the fact that relay partitioning and matched-filtering is capacity achieving only up to an $O(1)$ -term (even in the perfectly synchronous case). Combining (27) and (30), the proof of Theorem 1 is concluded. \square

In the derivation of Theorem 1 it was assumed that the relays have perfect knowledge of their assigned backward and forward channel's amplitude and phase fading realizations. Straightforward modifications of the proof of Theorem 1 show, however, that fading amplitude information is not required to obtain the scaling law (14). The essential component needed to achieve distributed orthogonalization is phase coherence and hence knowledge of the phase

fading realization. We shall next show that distributed orthogonalization can be achieved even if the relays can not realize full phase coherence due to synchronization errors.

Scenario 2: Relay terminal $\mathcal{R}_k \in \mathcal{X}_l$ performs matched-filtering of r_k w.r.t. the synchronization error uncompensated backward and forward channels $h_{k,l}$ and $g_{l,k}$, respectively. Appropriate normalization ensures that the average per-relay transmit power constraint is satisfied. This setup corresponds to a scenario where the backward and forward channels are learned (i.e., estimated) with the network operating in a perfectly synchronous mode, and synchronicity is subsequently lost. We call this mode of operation the “synchronization error uncompensated matched-filtering mode”.

Theorem 2. *For an asynchronous relay network operating in the synchronization error uncompensated matched-filtering mode, assuming a fixed number of source-destination terminal pairs L , in the large relay limit $K \rightarrow \infty$ such that⁷ $|\mathcal{X}_1| = |\mathcal{X}_2| = \dots = |\mathcal{X}_L| = K/L$, the network capacity scales as*

$$C = \frac{LT_s}{\Delta_1 + \Delta_2} \log(K) + O(1) \quad (31)$$

if

$$\mathcal{E}\{f(\xi)\} \neq 0 \quad \text{and} \quad \mathcal{E}\{p(\eta)\} \neq 0. \quad (32)$$

Moreover, network capacity is achieved with independent decoding at the destination terminals assuming that each \mathcal{D}_l has perfect knowledge of the effective scalar channels between the \mathcal{S}_i ($i = 1, 2, \dots, L$) and \mathcal{D}_l .

Proof: In contrast to Theorem 1 above, we perform matched-filtering w.r.t. the physical channels $h_{k,l}$ and $g_{l,k}$. The key ingredients of the proof, however, are identical to those of the proof of Theorem 1. For the sake of brevity, we shall therefore summarize the key steps only. The received signal at a destination terminal can be decomposed into desired signal, interference components

and a noise term according to (17) with

$$\begin{aligned}
d_{k,l} &= \frac{\sqrt{P_{l,k}}\sqrt{E_{k,l}}|g_{l,k}|p(\eta_{l,k})|h_{k,l}|^2 f(\xi_{k,l})}{\sqrt{2E_{k,l}|f(\xi_{k,l})|^2 + \sum_{i=1, i \neq l}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0}} \quad (33) \\
f_{k,l,m} &= \frac{\sqrt{P_{l,k}}\sqrt{E_{k,l}}\bar{g}_{l,k}g_{m,k}^*h_{k,m}^*\bar{h}_{k,l}}{|g_{m,k}|\sqrt{2E_{k,m}|f(\xi_{k,m})|^2 + \sum_{i=1, i \neq m}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0}} \\
x_{k,l,j} &= \frac{\sqrt{P_{l,k}}\sqrt{E_{k,j}}|g_{l,k}|h_{k,l}^*\bar{h}_{k,j}p(\eta_{l,k})}{\sqrt{2E_{k,l}|f(\xi_{k,l})|^2 + \sum_{i=1, i \neq l}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0}} \\
w_{k,l,j} &= \frac{\sqrt{P_{l,k}}\sqrt{E_{k,j}}\bar{g}_{l,k}g_{j,k}^*|h_{k,j}|^2 f(\xi_{k,j})}{|g_{j,k}|\sqrt{2E_{k,j}|f(\xi_{k,j})|^2 + \sum_{i=1, i \neq j}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0}} \\
v_{k,l,j,m} &= \frac{\sqrt{P_{l,k}}\sqrt{E_{k,j}}\bar{g}_{l,k}g_{m,k}^*h_{k,m}^*\bar{h}_{k,j}}{|g_{m,k}|\sqrt{2E_{k,m}|f(\xi_{k,m})|^2 + \sum_{i=1, i \neq l}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0}}
\end{aligned}$$

with the variances of the forwarded noise terms given by

$$a_{k,l} = \frac{P_{l,k}|\bar{g}_{l,k}|^2|h_{k,l}|^2}{2E_{k,l}|f(\xi_{k,l})|^2 + \sum_{i=1, i \neq l}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0} \quad (34)$$

$$b_{k,l,m} = \frac{P_{l,k}|h_{k,m}|^2|\bar{g}_{l,k}|^2}{2E_{k,m}|f(\xi_{k,m})|^2 + \sum_{i=1, i \neq l}^L E_{k,i}|f(\xi_{k,i})|^2 + N_0}. \quad (35)$$

Since $\mathcal{E}\{x_{k,l,j}\} = \mathcal{E}\{w_{k,l,j}\} = \mathcal{E}\{v_{k,l,j,m}\} = 0 \forall j, k, l, m$, it follows from the proof of Theorem 1 that asymptotically in K , the mutual information for the l -th source-destination terminal pair can be written as

$$I_l \xrightarrow{\text{w.p.1}} \frac{T_s}{\Delta_1 + \Delta_2} \log \left(1 + \frac{K|\delta_l|^2}{LN_0 \left(\frac{L}{K} + \alpha_l + \beta_l \right)} \right), \quad (36)$$

where δ_l , α_l and β_l are defined as in (24), (25) and (26) with $d_{k,l}$, $a_{k,l}$ and $b_{k,l,m}$ given by (33), (34) and (35), respectively. Using the same line of reasoning as in the proof of Theorem 1, it follows that $\delta_l \neq 0$ if

$$\mathcal{E}\{f(\xi)\} \neq 0 \quad \text{and} \quad \mathcal{E}\{p(\eta)\} \neq 0. \quad (37)$$

The strict positivity of α_l and β_l follows from (37) through Jensen's inequality [27] which ensures

$$\mathcal{E}\{|p(\eta)|^2\} \geq (\mathcal{E}\{|p(\eta)|\})^2 \geq |\mathcal{E}\{p(\eta)\}|^2.$$

Finally, using the fact that the cut-set upper bound for the perfectly synchronized case provides an upper bound for the synchronization error uncompensated asynchronous case, the proof of Theorem 2 is completed. \square

The results in Theorems 1 and 2 above can now be interpreted as follows:

- Irrespectively of whether the synchronization error is compensated or not, network capacity continues to exhibit the same scaling behavior as in the perfectly synchronous case (pre-log of L and logarithmic scaling in K). The presence of synchronization errors leads, however, to an SNR degradation at the destination terminals reflected by a reduced $O(1)$ -term in (14). Equivalently, the SNR degradation at the destination terminals leads to reduced network capacity (compared to the perfectly synchronized case) for a given number of relay terminals. Depending on the pulse shaping function $u(t)$ and the timing offset characteristics, this SNR degradation can easily become significant.
- The use of guard periods causes a reduction of the spatial multiplexing gain from $\frac{L}{2}$ to $\frac{LT_s}{\Delta_1 + \Delta_2} < \frac{L}{2}$. For large timing offsets, i.e., $(\Delta_1 + \Delta_2) \gg 2T_s$, this reduction can be significant.
- For the synchronization error uncompensated case, condition (32) essentially amounts to having a minimum level of residual coherence so that distributed orthogonalization is possible. Satisfying (32) requires that the timing offsets are small compared to the carrier frequency. For $\int_0^{T_s} u(t)u(t + \xi_{k,l}) dt \approx 1$ and $\int_0^{T_s} u(t)u(t + \eta_{l,k}) dt \approx 1 \forall l, k$ and $\omega_c \xi_{k,l}$ and $\omega_c \eta_{l,k}$ uniformly distributed in $[0, 2\pi)$, we have $\mathcal{E}\{e^{j\omega_c \xi_{k,l}}\} = 0$, $\mathcal{E}\{e^{j\omega_c \eta_{l,k}}\} = 0 \forall l, k$ and hence condition (32) will be violated. We can therefore conclude that for high carrier frequencies the synchronization requirements will be very strict.
- In the synchronization error compensated mode the condition (15) on the timing offsets essentially ensures that on average each relay participates in the relaying process. Phase coherence is not an issue here since the relays perform phase compensation.
- Comparing (18), (19), (20) with (33), (34), (35), we can see that (19) and (34) are equal and (20) and (35) are equal, which implies that the penalty to be paid for not compensating

the synchronization error is a reduction in the effective receive SNR at the \mathcal{D}_l quantified by the differences in the numerator of (18) and (33). Qualitatively, the corresponding SNR degradation is proportional to the amount of phase incoherence as measured by the ratio $\frac{(\mathcal{E}\{|f(\xi)||p(\eta)|\})^2}{|\mathcal{E}\{f(\xi)p(\eta)\}|^2}$ and can be significant as demonstrated in Simulation Example 1 in Section VI.

Imperfect CSI at the relay terminals. It is clear that distributed orthogonalization requires CSI at the relay terminals. As already outlined in the comments after the proof of Theorem 1, fading amplitude information is not required at the relays. Theorem 2 shows that distributed orthogonalization can be achieved in the presence of phase uncertainty at the relays provided the residual coherence condition (32) is satisfied. We note that phase uncertainty at the relay terminals could also stem from inaccurate phase fading knowledge of the backward and forward channels. In summary, we have therefore established that the distributed orthogonalization effect is quite robust w.r.t. CSI at the relays. All we need is a minimum amount of phase coherence at the relays such that the residual phase coherence condition (32) is satisfied. However, as already outlined above inaccurate phase information at the relays will result in an SNR degradation at the destination terminals.

Distributed orthogonalization in the AWGN case. Distributed orthogonalization as described in this section explicitly exploits the phase fading nature of the individual channels in the network to establish phase coherence between the individual source-destination terminal pairs and cancel multi-stream interference (through phase incoherence) in a distributed fashion. We note, however, that the concept of distributed orthogonalization is not restricted to fading channels. If the individual channels in the network are AWGN and the presence of different propagation delays is accounted for through multiplicative unit-modulus channel gains (narrowband assumption), distributed orthogonalization can be achieved by performing relay partitioning and (partial) phase compensation at the relays. For the distributed orthogonalization effect to occur, we need a “sufficient amount of phase averaging” in each cluster \mathcal{X}_l ($l = 1, 2, \dots, L$) as the number of relays grows large; this condition replaces the independent fading assumption in the discussion

above.

B. The Noncoherent Network Case

In the noncoherent network case, described for perfect synchronization in Section III.B, the k -th relay terminal simply amplifies the signal received during the first time slot subject to a per-relay transmit power constraint and transmits the result during the second time slot. Our main result for the asynchronous noncoherent network case is summarized in

Theorem 3. *For an asynchronous noncoherent relay network with a fixed number of source-destination terminal pairs L , under joint decoding at the destination terminals \mathcal{D}_l and perfect knowledge of the effective MIMO channel between the \mathcal{S}_l and the \mathcal{D}_l , the capacity of the AF protocol, in the large K limit, satisfies*

$$C_{AF}^\infty = \frac{T_s}{\Delta_1 + \Delta_2} \mathcal{E} \left\{ \log \det \left(\mathbf{I}_L + \frac{\rho}{L} \mathbf{H}_w \mathbf{H}_w^H \right) \right\} \quad (38)$$

provided that $\mathcal{E}\{|f(\xi)|\} > 0$ and $\mathcal{E}\{|p(\eta)|\} > 0$. The effective signal-to-noise ratio ρ is given by

$$\rho = \frac{\mathcal{E} \left\{ \frac{L E_{k,m} |f(\xi_{k,m})|^2}{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0} \right\}}{N_0 \mathcal{E} \left\{ \frac{1}{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0} \right\}}. \quad (39)$$

Proof: A major part of the proof of this theorem follows the proof of Theorem 3 in [1] with modifications taking into account the lack of synchronization. We shall therefore summarize the key steps of the proof only and focus on the differences to [1] due to synchronization errors.

Denoting the signal received at the k -th relay terminal during the first time slot by r_k , the signal transmitted during the second time slot is given by

$$t_k = \frac{r_k}{\sqrt{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0}}, \quad k = 1, 2, \dots, K.$$

The composite input-output relation can now be written in vector-matrix notation as

$$\mathbf{y} = \sum_{k=1}^K \frac{\mathbf{\Lambda}_k \mathbf{g}_k \mathbf{h}_k^T \mathbf{\Theta}_k}{\sqrt{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0}} \mathbf{s} + \sum_{k=1}^K \frac{\mathbf{\Lambda}_k \mathbf{g}_k n_k}{\sqrt{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0}} + \mathbf{z}$$

where n_k was defined in (5) and

$$\begin{aligned}\mathbf{s} &= [s_1 \ s_2 \ \dots \ s_L]^T, \quad \mathbf{z} = [z_1 \ z_2 \ \dots \ z_L]^T \\ \mathbf{h}_k &= [h_{k,1} \ h_{k,2} \ \dots \ h_{k,L}]^T, \quad \mathbf{g}_k = [g_{1,k} \ g_{2,k} \ \dots \ g_{L,k}]^T \\ \mathbf{\Lambda}_k &= \text{diag} \left\{ \sqrt{P_{l,k}} p(\eta_{l,k}) \right\}_{l=1}^L, \quad \mathbf{\Theta}_k = \text{diag} \left\{ \sqrt{E_{k,l}} f(\xi_{k,l}) \right\}_{l=1}^L.\end{aligned}$$

Since $I(\mathbf{y}; \mathbf{s}) = I(\mathbf{y}/\sqrt{K}; \mathbf{s})$, using the same arguments as in the proof of Theorem 3 in [1], we apply the central limit theorem to

$$\frac{\mathbf{y}}{\sqrt{K}} = \underbrace{\frac{1}{\sqrt{K}} \sum_{k=1}^K \frac{\mathbf{\Lambda}_k \mathbf{g}_k \mathbf{h}_k^T \mathbf{\Theta}_k}{\sqrt{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0}}}_{\mathbf{a}} \mathbf{s} + \underbrace{\frac{1}{\sqrt{K}} \sum_{k=1}^K \frac{\mathbf{\Lambda}_k \mathbf{g}_k n_k}{\sqrt{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0}}}_{\mathbf{b}} + \frac{\mathbf{z}}{\sqrt{K}}.$$

For $K \rightarrow \infty$, we obtain the jointly complex Gaussian vectors $\text{vec}(\mathbf{A}) \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_A)$ and $\mathbf{b} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_b)$ with

$$\begin{aligned}\mathbf{R}_A &= \mathcal{E} \left\{ \frac{P_{m,k} |p(\eta_{m,k})|^2 E_{k,i} |f(\xi_{k,i})|^2}{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0} \right\} \mathbf{I}_{L^2} \\ \mathbf{R}_b &= N_0 \mathcal{E} \left\{ \frac{P_{m,k} |p(\eta_{m,k})|^2}{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0} \right\} \mathbf{I}_{L^2}.\end{aligned}$$

Next, noting that the matrices \mathbf{R}_A and \mathbf{R}_b are nonzero if $\mathcal{E}\{|f(\xi)|^2\} \geq (\mathcal{E}\{|f(\xi)|\})^2 > 0$ and $\mathcal{E}\{|p(\eta)|^2\} \geq (\mathcal{E}\{|p(\eta)|\})^2 > 0$ and using the fact that the receiver has perfect knowledge of the effective channel matrix \mathbf{A} , we can conclude that the AF network capacity in the asynchronous case is given by

$$C_{AF}^\infty = \frac{T_s}{\Delta_1 + \Delta_2} \mathcal{E} \left\{ \log \det \left(\mathbf{I}_L + \frac{\rho}{L} \mathbf{H}_w \mathbf{H}_w^H \right) \right\}$$

with

$$\rho = \frac{\mathcal{E} \left\{ \frac{L E_{k,m} |f(\xi_{k,m})|^2}{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0} \right\}}{N_0 \mathcal{E} \left\{ \frac{1}{\sum_{l=1}^L E_{k,l} |f(\xi_{k,l})|^2 + N_0} \right\}}.$$

□

Like in the perfectly synchronous case, the simple AF protocol turns the network into an $L \times L$ point-to-point MIMO Rayleigh fading link with Gaussian noise and effective SNR given

by (39), high effective SNR capacity¹⁰

$$C_{AF}^{\infty} = \frac{LT_s}{\Delta_1 + \Delta_2} \log(\rho) + O(1)$$

and hence the same spatial multiplexing gain as in the coherent asynchronous case. The absence of CSI at the relay terminals results in a lack of distributed array gain, reflected in (39) not depending on K . The effective $L \times L$ MIMO channel between the \mathcal{S}_l and the \mathcal{D}_l is not diagonal (as in the coherent case) so that joint decoding at the destination terminals is crucial to achieve capacity. From (39) we can furthermore conclude that loss of synchronicity and hence $|f(\xi)| < 1$ results in a reduction of the effective SNR. Finally, we note that the condition on the timing offset characteristics to be satisfied in the noncoherent case is the same as the condition to be satisfied in the synchronization error compensated mode. In the noncoherent network case the relays do not perform distributed orthogonalization so that phase coherence at the relay terminals is not an issue. We “only” need to ensure that on average each relay participates in the relaying process.

V. CAPACITY SCALING IN FREQUENCY-SELECTIVE FADING NETWORKS

In this section, we expand the scope of our results to include frequency-selective fading in the network. In the following, we first describe the modifications of the channel and signal model introduced in Section II necessary to account for frequency-selective fading.

We assume that each of the channels in the network has an impulse response that consists of U taps with transfer functions for the individual channels given by

$$H_{k,l}(e^{j2\pi\theta}) = \sum_{n=0}^{U-1} h_{k,l}[n] e^{-j2\pi n\theta}, \quad 0 \leq \theta < 1, \quad k = 1, 2, \dots, K, \quad l = 1, 2, \dots, L \quad (40)$$

$$G_{l,k}(e^{j2\pi\theta}) = \sum_{n=0}^{U-1} g_{l,k}[n] e^{-j2\pi n\theta}, \quad 0 \leq \theta < 1, \quad k = 1, 2, \dots, K, \quad l = 1, 2, \dots, L \quad (41)$$

¹⁰The $O(1)$ -notation refers to effective SNR.

with independent (across k, l, n) circularly symmetric complex Gaussian taps normalized such that $\sum_{n=0}^{U-1} \mathcal{E}\{|h_{k,l}[n]|^2\} = \sum_{n=0}^{U-1} \mathcal{E}\{|g_{l,k}[n]|^2\} = 1$ ($k = 1, 2, \dots, K, l = 1, 2, \dots, L$). Note that in general there will be a continuum of delays. The channel model (40) and (41) is derived from the assumption of having U resolvable paths, where $U = \lfloor W\tau \rfloor + 1$ with W and τ denoting the signal bandwidth and delay spread, respectively. We employ a *block fading* channel model, i.e., we assume that the channel remains constant for the duration of the *coherence time* (assumed equal for all channels) and then changes independently to a new realization. Coding is performed over an infinite number of independent channel uses.

In order to simplify the exposition, we shall use a cyclic signal model [28] in the remainder of this section. This model essentially replaces linear convolution by periodic convolution and occurs, for example, in Orthogonal Frequency Division Multiplexing (OFDM) systems [29], [30]. We stress, however, that our results are generally valid and not specific to the OFDM case. In the following, we shall employ OFDM terminology and denote the number of tones in the OFDM system by N (the corresponding time-domain signal model is N -periodic). The cyclic prefix (CP) containing a periodic extension of the transmit signal (and hence redundant information) is of length $U_{cp} \geq U$ so that the frequency-selective fading channel is decomposed into N parallel flat-fading channels indexed by $p = 0, 1, \dots, N - 1$. The loss in spectral efficiency due to the CP is given by $\frac{U_{cp}}{N+U_{cp}}$ and becomes negligible for $N \gg U_{cp}$.

The signal received at the k -th relay terminal on the p -th tone is given by

$$r_k[p] = \sum_{l=1}^L \sqrt{E_{k,l}} H_{k,l}[p] s_l[p] + n_k[p] \quad (42)$$

where $H_{k,l}[p] = \sum_{n=0}^{U-1} h_{k,l}[n] e^{-j2\pi n \frac{p}{N}}$ is the complex-valued channel gain corresponding to the p -th tone on the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ link, $s_l[p]$ denotes the data symbol transmitted by the l -th source terminal on the p -th tone (independent across l, p) and $n_k[p] \sim \mathcal{CN}(0, N_0)$ is i.i.d. additive noise (independent across k, p). The relay terminal processes the received signal on each tone to

produce a corresponding unit average¹¹ energy output $t_k[p]$, which is transmitted over the p -th tone on the forward link. The signal received at the k -th destination terminal on the p -th tone is given by

$$y_l[p] = \sum_{k=1}^K \sqrt{P_{l,k}} G_{l,k}[p] t_k[p] + z_l[p] \quad (43)$$

where $G_{l,k}[p] = \sum_{n=0}^{U-1} g_{l,k}[n] e^{-j2\pi n \frac{p}{N}}$ is the complex-valued channel gain corresponding to the p -th tone on the $\mathcal{R}_k \rightarrow \mathcal{S}_l$ link and $z_l[p] \sim \mathcal{CN}(0, N_0)$ is i.i.d. additive noise (independent across l, p). The assumptions on the $E_{k,l}$ and the $P_{l,k}$ are the same as in Section II. We note that transmission/reception in this setup does not have to be perfectly synchronous; differences in propagation delay are allowed as long as the length of the effective channels does not exceed the length of the CP, i.e., $U \leq U_{cp}$.

A. The Coherent Network Case

We partition the relay terminals into L subsets each of which is assigned to one of the L source-destination terminal pairs. For each tone $p = 0, 1, \dots, N - 1$, relay terminal $\mathcal{R}_k \in \mathcal{X}_l$ performs matched-filtering of $r_k[p]$ w.r.t. the backward channel $H_{k,l}[p]$ and the forward channel $G_{l,k}[p]$ with appropriate normalization to meet the average per-relay per-tone transmit power constraint.

Theorem 4. *For a coherent frequency-selective fading relay network, assuming a fixed number of source-destination terminal pairs L , in the large relay limit $K \rightarrow \infty$ such that⁷ $|\mathcal{X}_1| = |\mathcal{X}_2| = \dots = |\mathcal{X}_L| = K/L$, the network capacity scales as*

$$C = \frac{LN}{2(N + U_{cp})} \log(K) + O(1). \quad (44)$$

Moreover, network capacity is achieved with (spatially) independent decoding at the destination terminals assuming that each \mathcal{D}_l has perfect knowledge of the N effective scalar channels between the \mathcal{S}_i ($i = 1, 2, \dots, L$) and \mathcal{D}_l .

¹¹Averaged over the backward channels, data symbols and noise.

Proof: From (42) and (43) it is clear that the frequency-selective fading relay network is decomposed into N parallel frequency-flat fading relay sub-networks. Using the fact that the statistics of $H_{k,l}[p]$ and $G_{l,k}[p]$ are independent of p , i.e., $H_{k,l}[p] \sim \mathcal{CN}(0, 1)$ and $G_{l,k}[p] \sim \mathcal{CN}(0, 1)$ for $k = 1, 2, \dots, K$, $l = 1, 2, \dots, L$, $p = 0, 1, \dots, N - 1$, it follows immediately that the statistical properties of the N sub-networks are identical. The asymptotic (in K) capacity of any one of these sub-networks can be derived using the proof of Theorem 1. The overall asymptotic network capacity is then given by the asymptotic capacity of any of the N sub-networks multiplied by the $N/(N + U_{cp})$ penalty in spectral efficiency resulting from the use of the cyclic prefix. This concludes the proof. \square

Theorem 4 essentially says that frequency-selectivity does not have an impact on network capacity (up to the reduced pre-log due to the use of a CP) and relay partitioning and matched-filtering on a per-tone basis achieves network capacity by decoupling the network into L frequency-selective single-input single-output links. Note that matched-filtering on a per-tone basis is equivalent to performing matched-filtering in the time-domain. Finally, we observe that the individual sub-networks have correlated complex-valued channel gains, which does, however, not affect the scaling result since we consider ergodic capacity. The network outage capacity will in general depend on this correlation and hence the amount of frequency-diversity.

Impact of partial CSI. Acquiring perfect knowledge of the frequency-selective fading backward and forward channels assigned to a given relay may be difficult if the channel order is high, which is the case for large bandwidths. In the following, we shall briefly illustrate that knowledge of a single tap (or a subset of the taps) of the backward and the forward channel assigned to a given relay is sufficient to obtain the scaling law (44). For simplicity, we assume that the k -th relay has perfect knowledge of the first tap $h_{k,l}[0]$ of its assigned backward channel and the first tap $g_{l,k}[0]$ of the corresponding forward channel. Performing matched-filtering by simply matching w.r.t. to the length U impulse response $[h_{k,l}[0] \ 0 \ \dots \ 0]$ on the backward link and the length U impulse response $[g_{l,k}[0] \ 0 \ \dots \ 0]$ on the forward link yields terms of the form

$[|h_{k,l}[0]|^2 \ h_{k,l}[1]h_{k,l}^*[0] \ \dots \ h_{k,l}[U-1]h_{k,l}^*[0]]$ and $[|g_{l,k}[0]|^2 \ g_{l,k}[1]g_{l,k}^*[0] \ \dots \ g_{l,k}[U-1]g_{l,k}^*[0]]$. We recognize the coherent contribution in the first tap of both effective impulse responses. When the number of relays grows large these coherent contributions will add up constructively whereas the remaining taps will add up destructively. Using the same technique as in the proof of Theorem 1, it then follows easily that even though the relay terminals do not match the backward and forward channels perfectly, distributed orthogonalization will be achieved. If the relays know a subset of the taps of the backward and forward channels matched-filtering is performed w.r.t. to the impulse responses obtained by replacing the unknown taps by zeros. Likewise, if partial phase fading information is available such that the residual phase coherence condition (32) is satisfied on a per-tone basis, it follows immediately that distributed orthogonalization can be achieved.

Finally, we note that while distributed orthogonalization can be realized with knowledge of a subset of the backward and forward channels' taps (or partial phase information on a per-tone basis) in the relay terminals, the SNR at the destination terminals will be reduced (compared to the case where the relays have full CSI). The amount of the corresponding SNR degradation depends on the fraction of the total channel energy contained in the known taps. Simulation Example 2 provides a quantitative assessment of this statement.

B. The Noncoherent Network Case

In the noncoherent case, on a tone-by-tone basis, the k -th relay terminal simply amplifies the signal received during the first hop subject to a per-tone transmit power constraint and transmits the result during the second hop. This signaling strategy will be called per-tone AF. Our main result for the noncoherent frequency-selective fading case is summarized as follows:

Theorem 5. *For a noncoherent frequency-selective relay network with a fixed number of source-destination terminal pairs L , under joint decoding at the destination terminals \mathcal{D}_l and perfect*

knowledge of the effective MIMO frequency-selective fading channel¹² between the \mathcal{S}_l and the \mathcal{D}_l , the capacity of the per-tone AF protocol, in the large K limit, satisfies

$$C_{AF}^{\infty} = \frac{LN}{2(N + U_{cp})} \mathcal{E} \left\{ \log \det \left(\mathbf{I}_L + \frac{\rho}{L} \mathbf{H}_w \mathbf{H}_w^H \right) \right\} \quad (45)$$

with

$$\rho = \frac{\mathcal{E} \left\{ \frac{L E_{k,m}}{\sum_{l=1}^L E_{k,l} + N_0} \right\}}{N_0 \mathcal{E} \left\{ \frac{1}{\sum_{l=1}^L E_{k,l} + N_0} \right\}}. \quad (46)$$

Proof: The proof follows in a straightforward fashion from the proofs of Theorems 4 and 3. \square

Theorem 5 states that per-tone AF, asymptotically in K , turns the relay network into a frequency-selective fading point-to-point MIMO link with the same ergodic capacity as that achieved by AF in the frequency-flat fading case. Just like in the frequency-flat fading case the effective MIMO channel between the \mathcal{S}_l and the \mathcal{D}_l is not orthogonalized so that joint decoding at the \mathcal{D}_l is needed to achieve AF network capacity. Finally, we note that again since we are considering ergodic capacity C_{AF}^{∞} does not depend on the number of taps or the power delay profile of the effective channel [31], [25] and is a function of the total energy in the effective channel only.

VI. NUMERICAL RESULTS

In this section, we present numerical results quantifying the impact of coherence at the relays on network capacity.

Simulation Example 1. This example quantifies the impact of uncompensated synchronization errors on coherent network capacity. The system parameters are chosen as $L = 2$, $T_s = 10^{-3}$ s, $\Delta_1 = \Delta_2 = T_s + 0.5 \times 10^{-6}$ s and the $\xi_{k,l}$ and the $\eta_{l,k}$ are assumed to be independent uniformly distributed in the interval $[0, 0.25 \times 10^{-6}]$ s $\forall l, k$. Note that over this range of timing offsets any reasonably behaved pulse-shaping function will result in $\int_0^{T_s} u(t)u(t + \xi_{k,l}) dt \approx 1$ and

¹²This is, of course, equivalent to knowing the N effective parallel frequency-flat fading MIMO channels.

$\int_0^{T_s} u(t)u(t + \eta_{l,k}) dt \approx 1 \forall l, k$. Consequently, the presence of timing offsets causes only phase differences between the physical channels and their corresponding effective channels. Fig. 2 shows the network capacity (obtained through Monte Carlo simulation) as a function of K for the synchronization error uncompensated mode with $\omega_c = 2\pi \times 10^6$ rad/s and $\omega_c = 2\pi \times 3 \times 10^6$ rad/s, respectively. For reference, we show the corresponding network capacity for the perfectly synchronous case and the synchronization error compensated mode. As expected, in all four cases, network capacity C grows logarithmically in K on account of distributed array gain. The performance in the synchronization error compensated mode is independent of the carrier frequency. In the uncompensated mode, however, we observe a significant reduction in C with increasing carrier frequency which is due to the fact that the effective per-stream SNR at the \mathcal{D}_l decreases for increasing support of $\omega_c \xi_{k,l}$ and $\omega_c \eta_{l,k}$. Note that $\omega_c = 2\pi \times 10^6$ rad/s results in $\omega_c \xi_{k,l}$ and $\omega_c \eta_{l,k}$ being uniformly distributed in $[0, \pi/2]$ whereas for $\omega_c = 2\pi \times 3 \times 10^6$ rad/s the distribution is uniform in $[0, 3\pi/2]$.

Simulation Example 2. This simulation example quantifies the impact of knowing only a subset of the backward and forward channels' taps on coherent frequency-selective network capacity. We assume $L = 2$ and $U = 10$ with a uniform power delay profile so that the individual channel taps are i.i.d. $\mathcal{CN}(0, 1/10)$. The per-tone SNR was set to $E_{k,l}/N_0 = P_{l,k}/N_0 = 10$ dB $\forall k, l$. Ignoring the loss in spectral efficiency due to the use of a CP, Fig. 3 shows the network capacity obtained by relay-partitioning, matched-filtering and independent decoding at the \mathcal{D}_l assuming that all taps, 9 out of the 10 (accounting for 90% of the total energy in the channel) and 5 out of the 10 (accounting for 50% of the total energy in the channel) taps of the assigned backward and forward channels are known at each of the relay terminals. In the cases where only a subset of the taps is known matched-filtering (in the frequency-domain) w.r.t. to the impulse responses obtained by replacing the unknown taps by zeros is performed. We can see that in all three cases network capacity scales logarithmically in K indicating that knowledge of a subset of the taps is sufficient to achieve distributed orthogonalization. Moreover, we observe that

partial CSI (in terms of knowing a subset of the taps only) leads to significant SNR degradation which amounts to an increased number of relay terminals needed for a given network capacity requirement.

VII. CONCLUSION

We studied the impact of synchronization errors, imperfect CSI at the relays and frequency-selective fading on capacity scaling in wireless relay networks. It was demonstrated that, under quite general conditions on the timing offset characteristics and CSI at the relays, the capacity scaling laws in both the coherent and the noncoherent network cases are not affected by the lack of synchronicity and/or imperfect CSI at the relays. For the coherent case, it was furthermore shown that distributed orthogonalization is possible as long as a minimum amount of coherence at the relay terminals can be sustained. The presence of frequency-selective fading does not impact the network capacity scaling laws provided the relaying strategies are properly modified to explicitly account for the presence of frequency-selectivity. Moreover, it was demonstrated that in wideband networks knowledge of as little as one tap of the assigned backward and forward channel at each of the relays is sufficient to achieve distributed orthogonalization. While synchronization errors and/or imperfect CSI at the relays do not impact the network capacity scaling laws a degradation of the SNR at the destination terminals is incurred which amounts to an increased number of relay terminals needed for a given network capacity requirement.

We note that the capacity results for the noncoherent network case are not as strong as the results for the coherent case. In the noncoherent case, we imposed a particular signaling strategy, namely AF, whereas in the coherent case the cut-set bound provides the ultimate performance limit irrespectively of the signaling strategy employed. The impact of other than AF relaying functions on the capacity of AWGN relay channels has recently been studied in [32].

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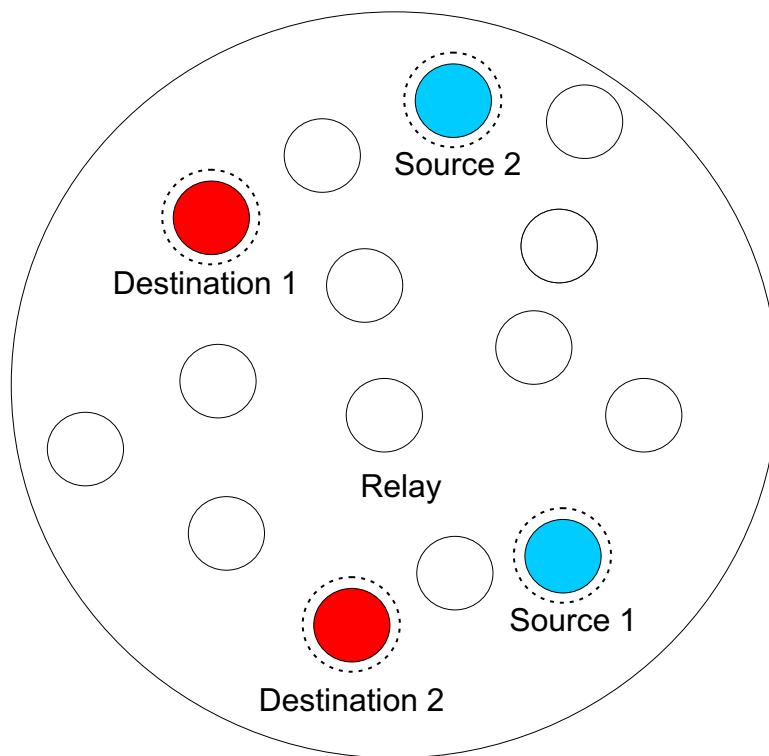


Fig. 1. *Schematic of a relay network with multiple source-destination terminal pairs.*

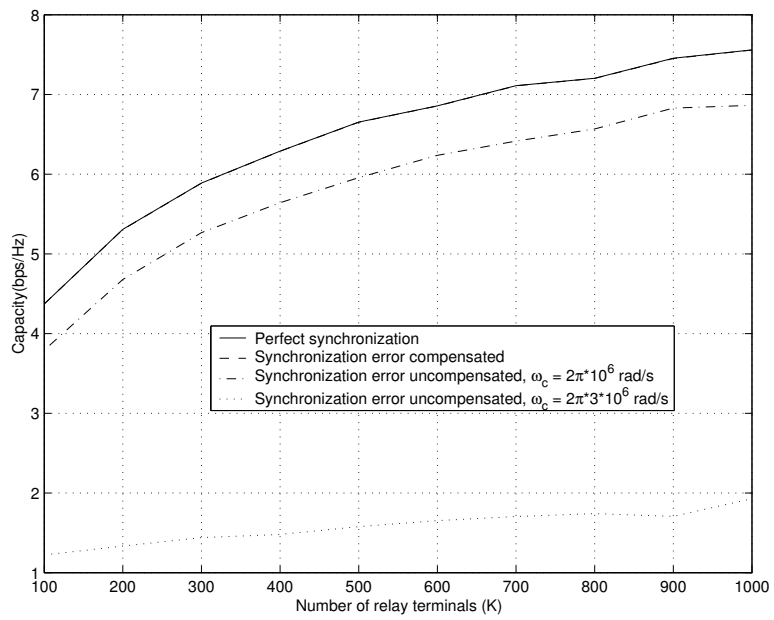


Fig. 2. Impact of synchronization errors on coherent network capacity as a function of K .

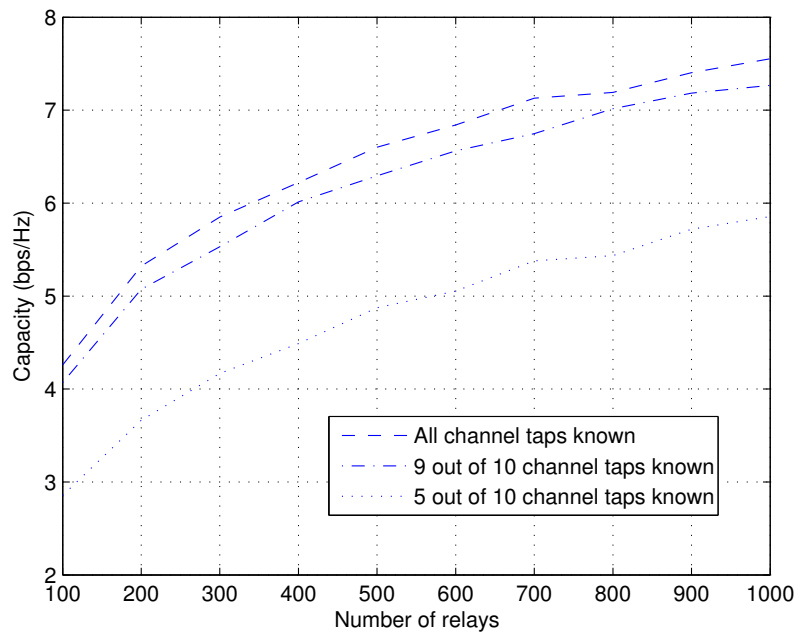


Fig. 3. *Impact of knowledge of subset of channel taps on coherent frequency-selective fading network capacity achieved by relay partitioning, matched-filtering at the relays and independent decoding at the destination terminals.*