Mathematical Methods for Machine Learning and Signal Processing SS 2019

Exam topics

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0 Homeworks

Very good knowledge of all material covered in homework problems will be assumed. This includes all problems, also the ones we did not cover in the discussion sections. The best way to obtain the necessary knowledge is to solve homework problems yourself.

1 Frame theory

1. Signal expansions in finite-dimensional Hilbert Spaces: orthonormal bases, biorthonormal bases, overcomplete expansions.
2. Characterization of general left-inverse matrix. Proof.
3. Frame for general Hilbert spaces. Upper and lower frame bounds. Meaning of the frame bounds. Examples.
4. The frame operator, its properties. Connection between the frame bounds and the spectral values of the frame operator.
5. Canonical dual frame. Connection between the frame, its canonical dual, and frame operator.
6. Signal expansions into a dual pair. General left-inverse operator.
7. Tight frame. Definition. Examples. Frame operator. Canonical dual.
8. Exact frames and bi-orthogonality. Be able to explain steps in the derivations, but no need to memorize the proofs.
9. Sampling theorem.
10. Sampling theorem as a frame expansion.
11. Design freedom in oversampled A/D conversion. Connection to left-inverse operator.
12. Noise reduction in in oversampled A/D conversion.
13. Weyl-Heisenberg frames
14. Wavelet frames

2 Compressed sensing – deterministic results

15. Signal separation as compressed sensing problem.
16. The set of sparse vectors as a union of subspaces.
17. Spark and P0 sparse signal recovery algorithm. Proof.
18. Coherence and P0 sparse signal recovery algorithm. Proof.
19. Basis pursuit recovery algorithm. Geometric intuition.
20. The null space property and sparse signal recovery guarantee for basis pursuit. Proof.
21. Low coherence implies null space property.
22. Uncertainty principles and sparse signal recovery for pairs of orthonormal bases. Proof.
23. The square-root bottleneck.

3 Matching pursuit and orthogonal matching pursuit

24. Matching pursuit (MP).
25. Orthogonal matching pursuit (OMP).
26. Difference between MP and OMP. Intuition. No need to memorize the proofs.

4 Super-resolution of positive sources

27. Super-resolution and sparsity. Mathematical model. Low-pass projection and its spectrum.
28. Stability of sparse recovery in the presence of noise. Super-resolution factor.
29. Dual certificate and duality lemma. Proof.
30. Simplest dual certificate for super-resolution. Proof of sparse signal recovery result in the noiseless case.

5 Randomized sampling

31. Restricted Isometry Property (RIP) and its meaning.
32. Be able to explain all steps on the proof of success of l1 minimization sparse signal recovery algorithm for measurement matrices that satisfy RIP. No need to memorize the proof.
33. The Johnson-Lindenstrauss lemma. Intuitive explanation.
34. Basic concentration inequalities. Proof via the moment-generating function.
35. Understand connection between concentration inequalities and RIP. No need to memorize the proof.

6 Principal Component Analysis

36. Singular value decomposition. Proofs.
37. Dimensionality reduction and SVD. Proofs.
38. Principal component analysis. Proofs.
39. Link between SVD and PCA. Proofs.

7 Matrix Completion

40. Matrix completion problem. Number of observation vs. number of degrees of freedom. Why the low-rand approximation makes sense.
41. Examples of low-rank matrices that cannot be recovered from few randomly located measurements.
42. Nuclear norm. Shape of nuclear norm ball. Derivation.
43. Coherence in matrix completion. How does low coherence excludes unrecoverable matrices.
44. Nuclear norm minimization problem and its similarity to l1 minimization problem.
45. Frobenius norm. Comparison to nuclear norm.
46. Operator norm for matrices and for general operators.

8 Scattering transform

47. Signal representations for pattern recognition. Desirable properties. Invariance to group operations (translation, rotation, etc). Stability to deformations.
48. Fourier modulus. Invariance to translations. Instability to deformations. Proofs.
49. Wavelets and averaging of high-frequency information.
50. Scattering transform and its structure.
51. Scattering transform for images. Scattering transform with 1 and 2 layers. Structure and meaning of the coefficients.
52. Understand formulations of invariance and stability results for scattering transform. No need to memorize the results but be able to explain them.